

Neutrino self-energy in a magnetized charge-symmetric medium

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Abstract. In this talk we present the calculation of the neutrino self-energy in presence of a magnetized medium. The magnetized medium consists of electrons, positrons, neutrinos and a uniform classical magnetic field. The background magnetic field is assumed to be weak compared to the W -Boson mass as a consequence of which only linear order corrections in the field are included in the W boson propagator. The electron propagator consists all order corrections in the background field. Our calculation is specifically suited for situations where the background plasma may be CP symmetric.

Keywords: Neutrino self-energy, Magnetic fields, Electron-positron plasma

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INTRODUCTION

In the present talk we assume the magnetic field to be less than the critical field corresponding the W^\pm bosons and consequently we take only linear order, in the magnetic field strength, corrections to its propagator. The gauge bosons are assumed to be not in thermal equilibrium and so their thermal modifications are not used. It is seen that the neutrino self-energy to linear order in Fermi coupling, G_F vanishes when the number of particles equals the number of anti-particles in the plasma. In astrophysical cases this does not happen in general as at these temperatures rarely particle numbers equals anti-particle numbers. But in the early universe and probably in the GRB fireball the temperature were supposed to be very high and so at these places we can expect that the number of particles equaled that of the antiparticles, to a great extent. In these circumstances we show that only order G_F^2 contributions remain in the expression of the neutrino self-energy. We have worked in the unitary gauge and have not discussed about the gauge independence of the result, as it is noted that in such calculations the self-energy generally is dependent on the gauge choice but the dispersion relation is independent of the gauge [1, 2].

GENERAL EXPRESSION FOR THE NEUTRINO SELF-ENERGY IN A MAGNETIZED CHARGE SYMMETRIC MEDIUM

The effect of the medium is represented by the 4-velocity of its centre-of-mass u^μ which looks like:

$$u^\mu = (1, \mathbf{0}), \quad (1)$$

in the rest frame of the medium. The u^μ is normalized in such a way that,

$$u^\mu u_\mu = 1. \quad (2)$$

Likewise the effect of the magnetic field enters through the 4-vector b^μ which is defined in such a way that the frame in which the medium is at rest,

$$b^\mu = (0, \hat{\mathbf{b}}), \quad (3)$$

where we denote the magnetic field vector by $\mathcal{B}\hat{\mathbf{b}}$. The 4-vector b^μ is defined in such a way that,

$$b^\mu b_\mu = -1. \quad (4)$$

In this talk we take the background classical magnetic field vector to be along the z -axis and consequently $b^\mu = (0, 0, 0, 1)$. The most general form of the neutrino dispersion relation in the magnetized plasma is of the form:

$$\Sigma(k) = R \left(a_{\parallel} k_{\parallel}^{\mu} + a_{\perp} k_{\perp}^{\mu} + b u^{\mu} + c b^{\mu} \right) \gamma_{\mu} L, \quad (5)$$

where $k_{\parallel}^{\mu} = (k^0, k^3)$ and $k_{\perp}^{\mu} = (k^1, k^2)$. With the above form of the neutrino self-energy the dispersion relation comes out as:

$$(1 - a_{\parallel}) E_{\nu_{\ell}} = \pm \left[((1 - a_{\parallel}) k_3 + c)^2 + (1 - a_{\perp})^2 k_{\perp}^2 \right]^{1/2} + b. \quad (6)$$

The coefficients a , b and c are functions of k_{\parallel}^2 , k_{\perp}^2 , $k \cdot u$ and $k \cdot b$. The results of the calculation are written in the rest frame of the plasma where the magnetic field points in the z -direction of the coordinate system.

THE FORM OF THE NEUTRINO SELF-ENERGY

In this talk we will work in the unitary gauge and consequently the three diagrams corresponding to the neutrino self-energy are as given in Fig. 1. The one-loop neutrino self-energy in a magnetized medium is comprised of three pieces, one coming from the W -exchange diagram which we will call $\Sigma^W(k)$, one from the tadpole diagram which we will designate by $\Sigma^T(k)$ and one from the Z -exchange diagram which will be denoted by $\Sigma^Z(k)$. The total self-energy of the neutrino in a magnetized medium then becomes:

$$\Sigma(k) = \Sigma^W(k) + \Sigma^T(k) + \Sigma^Z(k). \quad (7)$$

Here

$$-i\Sigma^W(k) = \int \frac{d^4 p}{(2\pi)^4} \left(\frac{-ig}{\sqrt{2}} \right) \gamma_{\mu} L i S_{\ell}(p) \left(\frac{-ig}{\sqrt{2}} \right) \gamma_{\nu} L i W^{\mu\nu}(q), \quad (8)$$

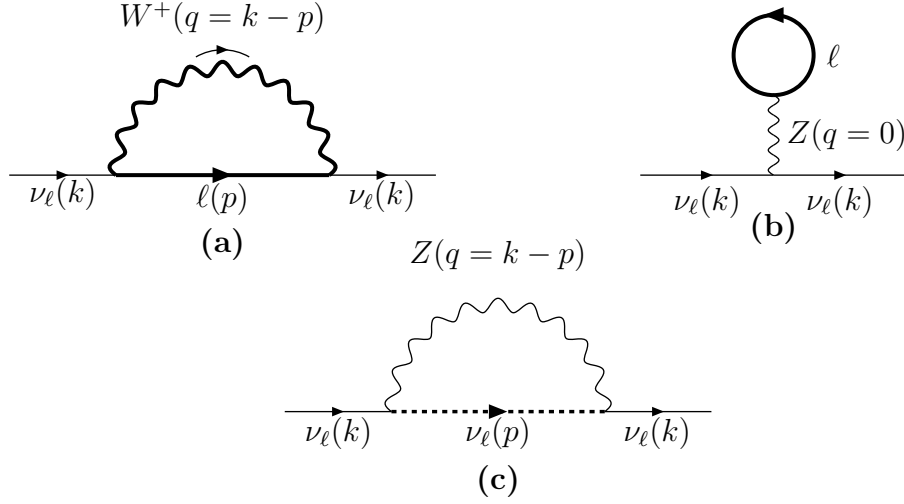


FIGURE 1. One-loop diagrams for neutrino self-energy in a magnetized medium. Diagrams (a) and (c) are the W and Z exchange diagrams and diagram (b) is the tadpole diagram. The heavy internal lines in diagrams (a) and (b) represent the W and the charged lepton propagators in a magnetized medium. The heavy dashed internal neutrino line in diagram (c) corresponds to the neutrino propagator in a thermal medium.

$$-i\Sigma^T(k) = -\left(\frac{g}{2\cos\theta_W}\right)^2 R \gamma_\mu iZ^{\mu\nu}(0) \int \frac{d^4p}{(2\pi)^4} \text{Tr}[\gamma_\nu (c_V + c_A \gamma_5) iS_\ell(p)], \quad (9)$$

and

$$-i\Sigma^Z(k) = \int \frac{d^4p}{(2\pi)^4} \left(\frac{-ig}{\sqrt{2}\cos\theta_W}\right) \gamma_\mu LiS_{\nu_\ell}(p) \left(\frac{-ig}{\sqrt{2}\cos\theta_W}\right) \gamma_\nu LiZ^{\mu\nu}(q). \quad (10)$$

In the above expressions g is the weak coupling constant and θ_W is the Weinberg angle. The quantities c_V and c_A are the vector and axial-vector coupling constants which come in the neutral-current interaction of electrons, protons (p), neutrons (n) and neutrinos with the Z boson. Their forms are as follows,

$$c_V = \begin{cases} -\frac{1}{2} + 2\sin^2\theta_W & e \\ \frac{1}{2} & \nu_e \\ \frac{1}{2} - 2\sin^2\theta_W & p \\ -\frac{1}{2} & n \end{cases}, \quad (11)$$

and

$$c_A = \begin{cases} -\frac{1}{2} & \nu, p \\ \frac{1}{2} & e, n \end{cases}. \quad (12)$$

Here $W^{\mu\nu}(q)$ and $S_\ell(p)$ stand for the W -boson propagator in a magnetic field and charged lepton propagator in presence of a magnetized plasma. The forms of the propagators are given in [3, 4, 5, 6, 7]. The $Z^{\mu\nu}(q)$ is the Z -boson propagator in vacuum and

$S_{\nu_\ell}(p)$ is the neutrino propagator in a thermal bath of neutrinos. The coefficients a, b, c are given as:

$$a = a_W + a_T + a_Z, \quad (13)$$

$$b = b_W + b_T + b_Z, \quad (14)$$

$$c = c_W + c_T + c_Z. \quad (15)$$

In Eq. (13) a_W, a_T and a_Z are composed of the parallel and perpendicular parts as shown in Eq. (5). For the case of a charge symmetric plasma, which perhaps existed in the early universe we have:

$$b = -\frac{4g^2k_0}{3M_W^2M_Z^2}\langle E_{\nu_\ell^B} \rangle N_{\nu_\ell} - \frac{2e\mathcal{B}g^2}{M_W^4} \int_0^\infty \frac{dp_3}{(2\pi)^2} \sum_{n=0}^\infty \sum_{\lambda=\pm 1} \left[\frac{k_3}{E_{\ell,n}} \left(p_3^2 + \frac{m_\ell^2}{2} \right) \delta_{\lambda,1}^{n,0} + k_0 E_{\ell,n} \right] f_{\ell,n}, \quad (16)$$

$$c = -\frac{2e\mathcal{B}g^2}{M_W^4} \int_0^\infty \frac{dp_3}{(2\pi)^2} \sum_{n=0}^\infty \sum_{\lambda=\pm 1} \left[k_0 \left(E_{\ell,n} - \frac{m_\ell^2}{E_{\ell,n}} \right) \delta_{\lambda,1}^{n,0} + \frac{k_3 p_3^2}{E_{\ell,n}} \right] f_{\ell,n}. \quad (17)$$

$$a_\perp = -\frac{2g^2e\mathcal{B}}{M_W^4} \int_0^\infty \frac{dp_3}{(2\pi)^2} \sum_{n=0}^\infty \sum_{\lambda=\pm 1} \left(\frac{\mathcal{H}}{2E_{\ell,n}} + \frac{m_\ell^2}{E_{\ell,n}} \right) f_{\ell,n} + \frac{g^2}{3M_W^4} \langle E_{\nu_\ell^B} \rangle N_{\nu_\ell}, \quad (18)$$

$$a_\parallel = -\frac{2g^2e\mathcal{B}}{M_W^4} \int_0^\infty \frac{dp_3}{(2\pi)^2} \sum_{n=0}^\infty \sum_{\lambda=\pm 1} \frac{m_\ell^2}{E_{\ell,n}} f_{\ell,n} + \frac{g^2}{3M_W^4} \langle E_{\nu_\ell^B} \rangle N_{\nu_\ell}. \quad (19)$$

In the above equations,

$$\mathcal{H} = e\mathcal{B}(2n+1-\lambda). \quad (20)$$

where n is a positive integer including zero and λ can take only two values ± 1 . The n corresponds to the Landau level number occurring in the energy of the charged leptons in a magnetic field and λ corresponds to the spin states of the leptons. The charged lepton wave-functions and the dispersion relation of them are calculated in [7, 8, 9]. More over,

$$f_{\ell,n} = \frac{1}{e^{\beta(E_{\ell,n}-\mu_\ell)} + 1}, \quad \bar{f}_{\ell,n} = \frac{1}{e^{\beta(E_{\ell,n}+\mu_\ell)} + 1}, \quad (21)$$

$$N_\ell = \frac{e\mathcal{B}}{2\pi^2} \sum_{n=0}^\infty \sum_{\lambda=\pm 1} \int_0^\infty dp_3 f_{\ell,n}, \quad \bar{N}_\ell = \frac{e\mathcal{B}}{2\pi^2} \sum_{n=0}^\infty \sum_{\lambda=\pm 1} \int_0^\infty dp_3 \bar{f}_{\ell,n}, \quad (22)$$

and N_ℓ^0 and \bar{N}_ℓ^0 corresponds to N_ℓ and \bar{N}_ℓ with $E_{\ell,n}$ in the distribution functions replaced by $E_{\ell,0}$, that is N_ℓ^0 and \bar{N}_ℓ^0 are the particle and anti-particle number densities in the lowest Landau level. The symbol $\delta_{\lambda,1}^{n,0} = 1$ only when $n = 0$ and $\lambda = 1$ and zero in other cases. N_{ν_ℓ} and \bar{N}_{ν_ℓ} are the number densities of the neutrinos and antineutrinos. Here $E_{\nu_\ell^B} = p \cdot u$ is the energy of the background neutrinos and $\langle E_{\nu_\ell^B} \rangle$ is the average energy per unit volume per neutrino in the heat bath. From the expressions of a, b, c we immediately

notice that all the contributions in the charge symmetric case are proportional to M_W^{-4} or G_F^2 . In addition to the charged leptons in the medium we can also have neutrons and protons in it, in that case the calculations will get extra contributions from diagrams where the protons replace the charged leptons and the neutrons replace the neutrino in the self-energy diagrams. In the $\mathcal{B} \rightarrow 0$ limit the last term on the right hand side of the above equation vanishes and we have,

$$b_{\mathcal{B} \rightarrow 0} = -\frac{16\sqrt{2}G_F k_0}{3M_Z^2} \langle E_{\nu_\ell}^{\mathcal{B}} \rangle N_{\nu_\ell} - \frac{16\sqrt{2}G_F k_0}{M_W^2} \langle E_\ell \rangle N_\ell. \quad (23)$$

The Eq. (23) resembles the results found in [10]. As the coefficients c and $a_\parallel - a_\perp$ are directly related to the existence of a non-zero magnetic field their zero field correspondence is not strictly permitted due to the non-perturbative nature of the Landau quantization of the charged fermions in presence of a magnetic field.

To order of g^2 the dispersion relation becomes,

$$E_{\nu_\ell} = [|\mathbf{k}|^2 - 2c\mathbf{k} \cdot \hat{\mathbf{b}} + 2(a_\parallel - a_\perp)k_\perp^2]^{1/2} + b, \quad (24)$$

where we have taken the positive sign of the square root in Eq. (6). The above dispersion relation can be simplified by binomially expanding the square root and neglecting terms of order more than g^2 . The expansion gives,

$$\begin{aligned} E_{\nu_\ell} &= |\mathbf{k}| - c\hat{\mathbf{k}} \cdot \hat{\mathbf{b}} + (a_\parallel - a_\perp) \frac{k_\perp^2}{|\mathbf{k}|} + b, \\ &= |\mathbf{k}| - c\cos\theta + (a_\parallel - a_\perp)|\mathbf{k}|\sin^2\theta + b, \end{aligned} \quad (25)$$

where $k^3 = k_z = |\mathbf{k}|\cos\theta$. The above equation implies that in presence of a magnetized medium the effective-potential acting on the neutrinos is of the form,

$$V_{\text{eff}} = b - c\cos\theta + (a_\parallel - a_\perp)|\mathbf{k}|\sin^2\theta. \quad (26)$$

From the expressions of a_\parallel and a_\perp in the **CP** symmetric case we see that in the lowest Landau level $a_\parallel - a_\perp$ is zero and in that case the effective potential is independent of a . With the form of the effective potential in Eq. (26) the problem of neutrino oscillations in the **CP** symmetric magnetized plasma in the early universe can be tackled. From Eq. (17) we see that the first term on the right hand side of the equation vanishes trivially in the zero field limit.

CONCLUSION

The talk was focussed on the calculation of the self-energy of the neutrino in a medium seeded with a uniform classical magnetic field. The calculations were carried out in the unitary gauge where the unphysical Higgs contribution does not appear. The background is supposed to be comprised of the charged leptons and neutrinos equilibrated at the same temperature. The magnitude of the magnetic field is such that only linear

contributions of the field appear in the charged W^\pm boson propagators but all orders of the field are present in the charged lepton propagators. In the present calculation we have assumed that the neutrinos are also in a state of thermal equilibrium. We explicitly write down the possible form of the neutrino self-energy in a magnetized medium by applying the concepts of Lorentz symmetry. The form of the self-energy becomes involved when the Lorentz breaking contributions are taken into account. The calculation is specially aimed at trying to find out the order G_F^2 contributions to the neutrino self-energy. This specific order of the coupling is important as we see that in a charge symmetric plasma the neutrino self-energy is proportional to G_F^2 only.

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